DOA Estimation Algorithm for Coherent Signals Based On Sparse Reconstruction

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Abstract: In this paper, an improved algorithm for direction of arrival(DOA) estimation of coherent signals is proposed from the point of view of signal sparse reconstruction. Through the study of sparse reconstruction theory, it is found that when there is interference between the atoms in the redundant dictionary, the accuracy of sparse signal reconstruction will be greatly affected. In order to reduce the interference between atoms, a new perception dictionary is designed and applied to DOA estimation of coherent signals in the compressed sampling matching pursuit (CoSaMP) algorithm. Through simulation, the running speed, DOA estimation and root mean square error of the estimated value of the proposed algorithm are verified and compared with the base tracking algori-thm.The simulation results verify the effectiveness of the proposed algorithm. The proposed algorithm has low computa-tional complexity, faster computational speed than the base pursuit algorithm, higher estimation accuracy than the orthogon-al matching pursuit (OMP) algorithm, and has the ability to overload the source.

1. Introduction

In the actual environment, signals are easily blocked by obstacles in the process of propagation, resulting in multipath propagation[1]. For multipath signals generated by the same source, they are coherent. The classical subspace algorithm MUSIC[2] algorithm and ESPRIT[3] algorithm need to carry out certain processing in the estimation of DOA of coherent signals, and they require a large amount of computation. With the development of the sparse signal reconstruction theory, the sparse reconstruction algorithm has better resolution and robustness than the conventional decoherence algorithm[4]. At present, the most common sparse reconstruction algorithms mainly include base pursuit[5](BP) algorithm and OMP[6] algorithm. BP algorithm has good stability in DOA estimation of coherent signals and can accurately reconstruct signals, but its complexity is high. The complexity of orthogonal matching tracking algorithm is much lower than BP algorithm, but the precision of signal reconstruction is much lower than BP algorithm.In view of this, Needell et al. proposed CoSaMP[7] algorithm, which has the advantages of the above two algorithms. The traditional CoSaMP algorithm is easily affected by interatomic interference in redundant dictionaries and cannot effectively reconstruct the coherent signals. In this paper, the adaptive perception dictionary to suppress interatomic interference is used to optimize the CoSaMP algorithm, and the optimized algorithm is used to process the DOA estimation of the coherent signals. The proposed algorithm is faster than BP algorithm, better than OMP algorithm, and has the ability of information overload.

2. Signal Model

It is assumed that the isotropic homogeneous uniform linear array[8] is composed of M elements, the distance between the elements is $d(d \le \lambda / 2)$, where λ is the signal waveleng-th. There are N far-field narrow-band signal sources from different directions. The direction of the signal source is denoted as $\theta_l(l = 1, \cdots, N)$. It is assumed that the power of noise is σ^2, and the signal $s(k)$ and noise $n(k)$ are zero-mean wide-stationary random processes[9]. The array output vector can be expressed as follows:

$$Y(k) = A(\theta)s(k) + n(k), \quad \text{k=1,} \cdots \text{,K} \tag{1}$$

Where, $A(\theta) = [a(\theta_1), a(\theta_2), \cdots, a(\theta_N)]$ is the array flow matrix, and $a(\theta_l)$ is the guide vector of the signal in direction θ_l.

$$a(\theta_l) = [1, e^{j2\pi d \sin(\theta_l)/\lambda}, \ldots, e^{j2\pi(M-1)d\sin(\theta_l)/\lambda}]^T \tag{2}$$

Where $l \in \{1, \ldots, N\}$. The multiple signals formed by the multipath propagation of the same signal in a signal source are called a coherent signal group.

The coherent signal group and the irrelevant signal constitute group L signal. The number of signals in each group is denoted as $b_1, b_2, \cdots, b_L(b_1 + b_2 + \cdots + b_L = N)$, and the original signal source is denoted as $s_l(k)$.

$d_i = [d_{i_1}, d_{i_2}, \cdots, d_{i_{b_i}}]^T (i = 1, 2, \cdots, L)$ contains the atte-nuation information of group i signals. When $d_i = 1$, group i is an irrelevant signal, otherwise it is a coherent signal group. In general, considering the additive white noise, the data vector of the k snap can be written as:

$$Y(k) = GS(k) + n(k), k = 1, \cdots, K \tag{3}$$

$G = [A_1 d_1, A_2 d_2, \cdots, A_L d_L] = [g_1, g_2, \cdots, g_L]$ is the generalized guidance matrix, A_i is the guidance vector matrix of the i coherent signal group, and the generalized guidance vector $g_i = \sum_{k=1}^{b_i} d_{i_k} a_{i_k} (i = 1, 2, \cdots, L)$, where the guidance vector of the k signal in the i coherent signal group is a_{i_k}, and the signal source vector $S(k) = [s_1(k), s_2(k), \cdots, s_L(k)]^T$. For the signal samples collected by the array at different times, they are denoted as $Y = [Y(1), \cdots, Y(K)]$, In the same way, S and N can be defined, so the signal received by the array can be expressed as:

$$Y = GS + N \tag{4}$$

3. The Design of the Perceptual Dictionary

Here we represent the perceptual dictionary as $W = [w_1, w_2, \cdots, w_N] \in C^{M \times N}$, and N is the number of angular samples. In order to make interatomic interference as small as possible, the adaptive function $W_n(n = 1, 2, \cdots, N)$ [10] to suppress interatomic interference is solved by using the optimization of adaptive minimum interference and undistorted response constraint, as follows:

$$\min w_n^H C_s C_s^H w_n \quad s.t. \quad A_n^H w_n = 1 \tag{5}$$

Where $C_s = AQ$, $Q = diag(\hat{s})$, $\hat{s} = | A^H y |$, the solution in the closed form is expressed as:

$$w_n = D_n a(\theta) \tag{6}$$

$$D_n = \frac{1}{a^H(\theta_n)(C_s C_s^H + \beta I_M)^{-1} a(\theta_n)} (C_s C_s^H + \beta I_M)^{-1} \tag{7}$$

$n = 1, \cdots, N$, β is the regularization parameter.

In addition, $C_s C_s^H = AQQ^H A^H = AU_t A^H$, the above equation can also be written as:

$$D_n = \frac{1}{a^H(\theta_n)(AU_t A^H + \beta I_M)^{-1} a(\theta_n)} (AU_t A^H + \beta I_M)^{-1} \tag{8}$$

4. Implementation Steps of the Algorithm

DOA of irrelevant signals is estimated using MUSIC algorithm, and its pseudo-power spectrum is $P_{MUSIC}(\theta) = \frac{1}{\| a^H(\theta) U_n \|^2}$. The improved CoSaMP algorithm is used to estimate the DOA of coherent signals.Combined with the designed adaptive perception dictionary W and CoSaMP algorithm(W-CoSaMP), the implementation steps of the algorithm are summarized as follows:

The input: Measurement matrix A, observation vector y, sparsity K;

Output: index set Λ, reconstruction signal vector $\hat{x} = u$;

(1) Initialization: the redundancy error is set as: $r_0 = y$, $\Lambda_0 = \varnothing$, $t = 1$.

(2) $y_u = A^\dagger y$, Operator $(\cdot)^\dagger$ represents Moore-Penrose inverse, for $n = 1, 2, \cdots, N$, to calculate $\hat{y}_t = |W^H r_{t-1}|$; $U_t = diag\left(|\hat{y}_u|^4\right)$.

$D_n = \frac{1}{a^H(\theta_n)(AU_t A^H + \beta I_M)^{-1} a(\theta_n)} (AU_t A^H + \beta I_M)^{-1}$; $w_n = D_n a(\theta_n)$; $W = [w_1, w_2, \cdots, w_N]$; $\hat{y}_u = |W^H y|$.

(3) $2K$ maximum values are selected from $\hat{y}_t$, and these values correspond to; column number j of A to form set J_0.

(4) Update index set $\Lambda_t = \Lambda_{t-1} \cup J_0$; For all the $j \in J_0$; $A_t = A_{t-1} \cup a_j$.

(5) Find the least squares solution of $y = A_t u_t$:

$$\hat{u}_t = \arg\min_{u_t} \|y - A_t u_t\| = \left(A_t^T A_t\right)^{-1} A_t^T y$$
.

(6) From $\hat{u}_t$, the item with the largest absolute value K is denoted as $\hat{u}_{tK}$, the corresponding column K in A_t is denoted as A_{tK}, the corresponding column sequence number of A is denoted as Λ_{tK}, Update index set $\Lambda_t = \Lambda_{tK}$.

(7) Update the residual:

$$r_t = y - A_{tK}\hat{u}_{tK} = y - A_{tK}\left(A_{tK}^T A_{tK}\right)^{-1} A_{tK}^T y$$

(8) $t = t+1$, if $t > S$, stop the iteration; Otherwise, go back to step (2) and continue the iteration.

When the DOA of coherent signals is estimated, the sparsity K is priori information. As the number of measured values is generally selected as $O(K \log N)$, the sparsity can be roughly estimated as $M / \log N$. In order to accurately reconstruct the desired signal, let $K = M$, $y_c = A_{tK}\hat{u}_{tK}$. Define the $M \times 1$ dimensional vector q:

$$q(i) = \frac{y_c(\theta_i)}{\max\limits_{i\in\{1,\cdots,M\}} y_c(\theta_i)}, \quad i=1,\cdots, M \tag{9}$$

In practice, if $q(i) > \eta_u$, where η_u is the selected threshold, then θ_i is the true DOA of the coherent signal; otherwise, θ_i is the wrong DOA. $a(\theta_i)(i \in 1,\cdots,M)$ in column i of the dictionary $\hat{A}_t$ corresponding to direction θ_i is the DOA estimation of the coherent signal.

5. The Experimental Simulation

There are M elements in the uniform linear array, the distance between the elements is half wavelength, and the number of fast beats is denoted as N. The direction range selected by the super complete dictionary is $[0^\circ, 180^\circ]$, and samples are evenly spaced with 1° as the interval. The covariance matrix of the signal can be estimated as:

$$R_Y = \frac{1}{N}\sum_{t=1}^{N} Y(t)Y^H(t) \tag{10}$$

Suppose all the signals have the same energy σ_s^2. σ_n^2 is the energy of the noise. The SNR is $10\log 10(\sigma_s^2 / \sigma_n^2)$. The number of Monte Carlo experiments is N. Here, the root mean square error

(RMSE) estimated by DOA is used as the performance indicator of the algorithm. The root mean square error is defined as follows:

$$RMSE = \sqrt{\frac{1}{NK}\sum_{n=1}^{N}\sum_{k=1}^{K}(\hat{\theta}_k(n)-\theta_k)^2} \tag{11}$$

Where, $\hat{\theta}_k(n)$ is the estimation of real signal Angle θ_k in the n Monte Carlo simulation, and K is the number of signals.

In this paper, DOA estimation is simulated in the presence of a group of coherent signals, and BP algorithm is used to compare the DOA estimation of coherent signals. The number of elements is set to A. SNR=10dB. Suppose that the number of signal sources received by the array is five, among which there are two irrelevant signals whose direction is $[-50^{\circ},30^{\circ}]$, and the rest are three coherent signals whose direction is $[-22^{\circ},15^{\circ},57^{\circ}]$. The atten-uation coefficient of the coherent signal is : $[-0.5548+j*0.5762,-0.6574-j*0.2456,-0.8304-j*0.3287]$.

Figure 1 is the simulation result of DOA estimation of the signal. Figure 2 shows the simulation results of RMSE estimated by DOA under different SNR.

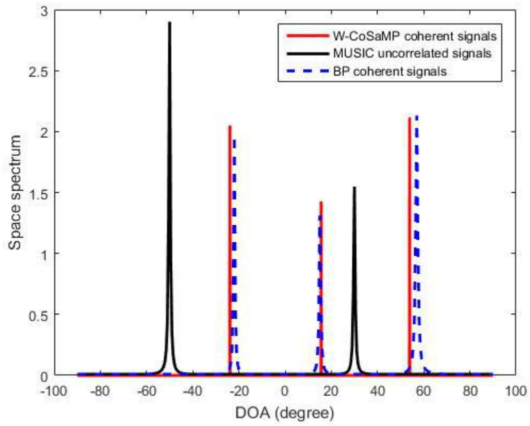

Figure 1: Comparison between the method in this paper and the DOA estimation of BP algorithm.

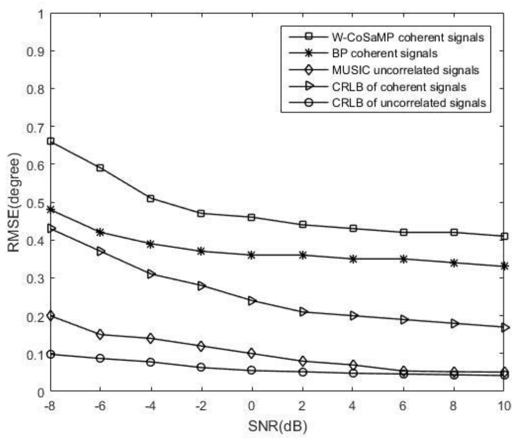

Figure 2: Comparison of RMSE for DOA estimation by different methods.

Through the analysis of simulation results, it can be concluded that in the sparse signal reconstruction algorithm, BP algorithm has more accurate signal estimation ability. The accuracy of signal estimation is close to that of BP algorithm, and the error is within acceptable range. In addition, the steps of the method in this paper are relatively simple, and it can be found from the simulation experiment that the operation speed of the method in this paper is obviously faster than that of BP algorithm. Because the method in this paper deals with the coherent signal and the irrelevant signal separately, it can estimate more signal sources.

6. Conclusions

In this paper, an algorithm for DOA estimation of coherent signals is proposed based on CoSaMP algorithm to suppress interatomic interference. The information of irrelevant signals is removed from the signal subspace and the information of coherent signals is distinguished by sparse reconstruction method. Simulation results show that the method presented in this paper has a faster speed than BP algorithm, and its DOA estimation error is within the allowable range, which can achieve relatively accurate DOA estimation. In addition, it can estimate more signal sources.

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